Dynamics of Cosmological Perturbations in Multi-Speed Inflation

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Abstract

We study a multi-field inflationary theory with separable Lagrangian, which has different speed of sound for each field. We find that the fields always coupled at perturbative level through gravitational interaction. We show that if the coupling terms among the perturbation fields are weak enough, these fields can be treated as a combination of decoupled fields, which are similar to normal modes in coupled oscillation. By virtue of such fields, the curvature perturbation at the horizon-crossing can be calculated up to the leading order of slow variation parameters via $\delta\mathcal{N}$ formalism. Explicitly, we consider a model of multi-speed DBI inflation, and calculate the power spectrum in detail. The result depends on the ratio of different speeds of sound, and shows an apparent amplification when the ratio deviates from unity.

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I. INTRODUCTION

Inflationary cosmology has become the prevalent paradigm to understand the early stage of our universe, with its advantages of resolving the flatness, homogeneity and monopole problems [1, 2], and predicting a approximately scale-invariant primordial power spectrum consistent with current cosmological observations [3] very well. However, a single field inflation model often suffers from fine tuning problems on the parameters of its potential, such as the mass and the coupling constant.

In recent years, people has noticed that, when a number of scalar fields are involved in the inflationary stage, they can relax many limits on the single scalar inflation model [4]. Usually, these fields are able to work cooperatively to give an inflationary stage long enough, even none of them can sustain inflation separately. Models of this type have been considered later in Refs. [5–8]. The main results showed that both the e-folding number $\mathcal{N}_e$ and the curvature perturbation $\mathcal{R}$ are approximately proportional to the number of the scalars N. Later, the model of N-flation was proposed by Dimopoulos *et al.* [9], which showed that a number of axions predicted by string theory can give rise to a radiatively stable inflation. This model has explored the possibility for an attractive embedding of multi-field inflation in string theory.

Over the past several years, based on the recent developments in string theory, there have been many studies on its applications to the early universe in inflationary cosmology. An interesting inflation model, which has a non-canonical kinetic term inspired by string theory, was studied intensively in the literature. Due to a non-canonical kinetic term, the propagation of field fluctuations in this model is characterized by a sound speed parameter and the perturbations get freezed not on Hubble radius, but on the sound horizon instead. One specific realization of this type of models can be described by a Dirac-Born-Infeld-like (DBI) action [10, 11]. Based on brane inflation [12], the model with a single DBI field was investigated in detail[13–15], which has explored a window of inflation models without flat potentials. In this model, a warping factor was applied to provide a speed limit which keeps the inflaton near the top of a potential even if the potential is steep.

Motivated by the effective description of multiple D-brane dynamics in string theory [16], an interesting inflation model involving multiple sound speeds with each sound speed characterizing one field fluctuation was proposed [17, 18, 34]. The authors of Refs.[17, 18] suggest this scenario can be realized by a number of general scalar fields with arbitrary kinetic forms, and these scalars have their own sound speeds respectively. Therefore, this model is dubbed as "*Multi-Speed Inflation*" (MSI). In this model, the propagations of field fluctuations are individual, and the usual conceptions in multi-field inflation models might be not suitable in this scenario. For example, in a usual generalized N-flation model, the length scale for perturbations being freezed takes the unique sound horizon; however, in our model it corresponds to the maximum sound horizon. It is worth emphasizing that the model of MSI is different from the usual DBI N-flation in which only multiple moduli fields are involved in one DBI action [19–25], but MSI is constructed by multiple Kessence type actions. For example, an explicit model of MSI is made of two DBI fields in Ref [17], and its primordial perturbations including curvature and entropy modes and their non-Gaussianities were considered.

In the current paper we extend the model into a much more generic case by relaxing the form of the Lagrangian. We find that even there are no coupling terms among the inflaton fields, their perturbation modes are coupled due to the nonlinear gravitational interaction.

We also find that if these couplings are weak, the independent components of the fields can be treated as a combination of normal modes, by which the curvature perturbation at the horizon-crossing can be calculated in detail up to leading order of slow variation parameters via $\delta\mathcal{N}$ formalism. As a specific example, we consider a model of multi-speed inflation involving two DBI fields. We show that in the relativistic limit, the coupling between two fields is mainly reflected by the damping terms in the perturbation equations. The difference between the sound speeds of these two fields is able to generate a considerable amplification on the amplitude of the primordial curvature perturbation. This is greatly different from the usual analysis on the inflationary model constructed by two canonical fields[27, 28].

The paper is organized as follows. In Section II, we study the cosmological perturbation theory of the MSI model in a generic case, then show that the inflaton fluctuations could be decoupled through a redefinition of fields at leading order under the assumption of weak couplings. In Section III, we focus on a model of MSI involving two fields, and study the detailed field transformation matrix to illustrate that the decoupling process of the field fluctuations is reliable under the weak coupling approximation. Specifically, we analyze a specific model constructed by two DBI fields and give its curvature perturbation. Section IV presents a summary and discussion.

In the paper we take the normalization $M_p^2 = 1/8\pi G = 1$ and the sign of metric is adopted as $(-,+,+,+)$ in the following.

II. THE MSI MODEL

An inflation model constructed with a single Kessence was originally proposed by [30] and later its perturbation theory was developed in [31]. In the literature this type of model has been widely studied, and one of the most significant features is that there is an effective sound speed describing the propagation of the perturbations. In the model of MSI, for each k-essence field there is one sound speed correspondingly. Therefore, the field fluctuations in this model do not propagate synchronously due to different sound speed parameters, but get coupled because of gravitational interactions. In the current paper, our main interests focus on the effects of multiple sound speeds and mode-couplings at the level of linear perturbation. Before studying the perturbations, we first take an investigation on the background equations.

A. The setup of background model

The action is the summation of Hilbert-Einstein action and the action of N different fields minimally coupled with gravity[17]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \sum_{I=1}^{N} P_I(X_I,\phi_I)\right], \qquad (1)$$

with

$$X_I = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi_I\nabla_\nu\phi_I. \qquad (2)$$

which is the kinetic term of the inflaton field ϕ_I. From now on the subscript of capital Latin letter $I, J, ...$ always denote the different inflaton fields. In this model, the Klein-Gordorn

equations in a homogeneous background are

$$\ddot{\phi}_I + \left(3H + \frac{\dot{P}_{I,X_I}}{P_{I,X_I}}\right)\dot{\phi}_I - \frac{P_{I,I}}{P_{I,X_I}} = 0. \tag{3}$$

Here and thereafter, the subscript ", I" denotes the derivative with respect to the field ϕ_I, and ", X_I" denotes the derivative with respect to X_I, for simplicity. Assuming a flat Friedman-Robertson-Walker background, the Friedman equations are given by

$$3H^2 = \rho = \sum_I (2X_I P_{I,X_I} - P_I), \tag{4}$$

$$\dot{H} = -\frac{1}{2}(\rho + \sum_I P_I) = -\sum_I X_I P_{I,X_I}, \tag{5}$$

where H denotes the Hubble parameter $\dot{a}/a$ at a given time.

To get analytical results, it is helpful to define some parameters to characterize the kinetic behavior of the fields. In single-field canonical inflation, a set of slow-roll parameters are necessary. To ensure their smallness is not only the requirement of inflation to occur and endure for long enough, but also some simplification in analytical calculation. In a generic case, just as in [32], we define a similar set of slow-variation parameters. Recall that in the simplest single-field inflation with canonical kinetic energy and flat potential, one can define two slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} \sim \frac{\dot{\phi}^2}{V(\phi)} \sim \left(\frac{V'(\phi)}{V(\phi)}\right)^2, \tag{6}$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} \sim \frac{\ddot{\phi}}{H\dot{\phi}} \sim \frac{V''(\phi)}{V(\phi)}. \tag{7}$$

Here the $\sim$ symbol denotes the approximate equality which only holds in slow-variant environment and neglect the possible numerical factor. The second equality defines the slow-roll parameter via the kinetic side, whereas the third equality defines it in the potential viewpoint. If one only deal with the parameters under slow-roll condition, for instance in a model with canonical kinetic terms, these definitions are almost equal, and assuming one set is small can immediately assure the smallness of the other sets. This is just because the dynamic behavior simply relies on the shape of potential. But in a generic case, for instance in DBI or k-inflation model, the dependence may be much more complex and these different definitions are no more equivalent. Since they are all very important in parameterizing our final result, we have to define two sets of slow-variation parameters from different viewpoints. In general, we should replace the slow-variation parameters by some slow-variation matrixes that can reflect the correlations between different fields. Since we are dealing with a separable Lagrangian, we only need different parameters for each field. This is equivalent to the case with a diagonal slow-variation matrix. As is mentioned above, one way to trace the motion is to define the parameter by each kinetic term of one specific field:

$$\epsilon_I = \frac{P_{I,X_I} X_I}{H^2}, \tag{8}$$

$$\eta_I = \frac{\dot{\epsilon}_I}{H\epsilon}. \tag{9}$$

In DBI model the field may roll down the potential very fast, but the parameter ϵ_I defined here is still small and slowly variant. The summation of such parameters gives the original version of them in canonical case:

$$\epsilon = -\frac{\dot{H}}{H^2} = \sum_I \epsilon_I, \tag{10}$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = \sum_I \eta_I. \tag{11}$$

Note that the magnitude of ϵ_I and ϵ obeys $\epsilon_I/\epsilon \sim \mathcal{O}(1/N)$ in general. To accomplish our calculation we need another version of slow-variation parameters defined in the potential aspect:

$$e_I = -\frac{P_{I,I}}{3H^2}, \tag{12}$$

$$h_I = \frac{\dot{e}_I}{He_I}, \tag{13}$$

This is an analogue of $\epsilon \sim (V'/V)^2$, and for future convenience we take a squared root. It is not necessarily small in an infrared (IR) DBI model, in which the value of the inflaton field may be very small. So for convenience let us focus on the ultraviolet (UV) DBI model where ϕ moves from UV side of the potential to the IR side. We should remind us that e_I^2 has the same order with ϵ_I in a model with canonical kinetic energy, but is very close to 1 in multi-DBI model. We refer to it as a half-order slow variation parameter in the former case, and tried to preserve terms higher than e_I^2. Besides,

$$\iota_I = -\frac{P_{I,II}}{3H^2}. \tag{14}$$

as another extension of $\eta \sim V''/V$ should be defined. According to the speed of sound in the single-field case, we can define the speed parameter of each field:

$$c_{sI}^2 = \frac{P_{I,X_I}}{\rho_{I,X_I}} = \frac{P_{I,X_I}}{2X_I P_{I,X_I X_I} + P_{I,X_I}} \tag{15}$$

$$s_I = \frac{\dot{c}_{sI}}{Hc_{sI}} \tag{16}$$

By this definition, the sound speed is the actually the effective propagation speed of the perturbations, which is different from the adiabatic sound speed which is defined as $c_{s(ad)}^2 = \dot{P}/\dot{\rho}$ when the cosmic fluid is described by scalar fields[34]. Notice that in a multi-Dirac-Born-Infeld model we have an extra relationship $P_{I,X_I} = 1/c_{sI}$ which will connect this parameter with ϵ_I. In general, c_{sI} needs not to be the same for all the fields, which makes this model different from the ordinary multi-DBI model and may give us some new phenomenon that we are about to investigate. For an analysis of c_s as an inverse Lorentz vector in a single-field DBI inflation, see [29].

After having defined such parameters, the main quantities involving P_I can be expressed

in terms of these parameters directly from the inverse relations above. For instance,

$$
\begin{aligned}
P_{I,X_I} &= \frac{H^2}{X_I}\epsilon_I, &(17)\\
P_{I,X_IX_I} &= \frac{\epsilon_I}{2}\left(\frac{H}{X_I}\right)^2\left(\frac{1}{c_{sI}^2}-1\right), &(18)\\
P_{I,I} &= -3H^2 e_I, &(19)\\
P_{I,II} &= -3H^2\iota_I. &(20)
\end{aligned}
$$

Besides the P's, another important quantity is the time derivative of one field. By using the Klein-Gordon equation (3) we have

$$\frac{\dot{\phi}_I}{2H} = \frac{\epsilon_I}{e_I}\left(1-\frac{1}{3}\epsilon\frac{\eta_I}{\epsilon_I}-\frac{2}{3}\epsilon\frac{e_I}{h_I}\right). \tag{21}$$

B. Generic perturbation analysis

In this subsection we perform a generic analysis on the linear perturbations of the model introduced previously. Since we are working in the frame of a cosmological system, the metric perturbations ought to be included as well as the field fluctuations. We would like to expand the action to the second order by virtue of the Arnorwitt-Deser-Misner (ADM) formalism [35].

To start, the spacetime metric in the ADM formalism is written as,

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i+N^idt)(dx^j+N^jdt), \tag{22}$$

the action can reads

$$S = \frac{1}{2}\int dtd^3x\sqrt{h}N\left(\frac{1}{2}R^{(3)}+\sum_I P_I\right)+\frac{1}{2}\int dtd^3x\sqrt{h}N^{-1}\left(E_{ij}E^{ij}-E^2\right) \tag{23}$$

with $E_{ij}=(1/2)(\dot{h}_{ij}-\nabla_iN_j-\nabla_jN_i)$. When expanding the action up to the second order, we can decompose the fields and Lagrangian multipliers as

$$
\begin{aligned}
\phi_I(t,\mathbf{x}) &= \phi_I(t)+\delta\phi_I(t,\mathbf{x}), &(24)\\
N &= 1+\alpha, &(25)\\
N_i &= \partial_i\beta. &(26)
\end{aligned}
$$

First, the primary Hamiltonian and the momentum constrains gives two relations

$$
\begin{aligned}
\alpha &= \frac{1}{2H}\sum_I P_{I,X_I}\dot{\phi}_I\delta\phi_I, &(27)\\
\partial^2\beta &= \frac{1}{2H}\sum_{IJ}\left\{-\frac{P_{I,X_I}}{c_{sI}^2}\dot{\phi}_I\dot{\delta\phi}_I+(P_{I,I}-2X_IP_{I,IX_I})\delta\phi_I\right.\\
&+\left.\frac{P_{I,X_I}}{H}\left(\frac{X_IP_{J,X_J}}{c_{sI}^2}\dot{\phi}_J\delta\phi_J-3H^2\dot{\phi}_I\delta\phi_I\right)\right\}.
\end{aligned}
$$

Then we can make the perturbative expansion of the action up to second order by using the Lagrangian constraints:

$$\begin{aligned}
S_2 &= \int dtd^3x\frac{a^3}{2}\left\{P_{I,X_IX_I}v_I^2 + P_{I,X_I}\dot{\delta\phi_I}^2 - \frac{1}{a^2}P_{I,X_I}\partial_i\delta\phi_I\partial_i\delta\phi_I - \frac{3}{2}\dot{\phi}_I\dot{\phi}_J P_{I,X_I}P_{J,X_J}\delta\phi_I\delta\phi_J\right.\\
&+ \frac{1}{H}P_{I,X_I}\dot{\phi}_I\delta\phi_I\left(P_{J,X_J}v_J + P_{J,J}\delta\phi_J\right) + P_{I,II}\delta\phi_I^2 + 2P_{I,IX_I}v_I\delta\phi_I\\
&+ \left. P_{I,X_I}\left[3\dot{\phi}_I^2\left(\frac{1}{4H^2}P_{K,X_K}P_{L,X_L}\dot{\phi}_K\dot{\phi}_L\delta\phi_K\delta\phi_L\right) - \frac{2}{H}\left(P_{J,X_J}\dot{\phi}_J\delta\phi_J\right)\dot{\phi}_I\dot{\delta\phi}_I\right]\right\}, \qquad (28)
\end{aligned}$$

with

$$v_I = \dot{\phi}_I\dot{\delta\phi}_I - \left(\frac{1}{2H}P_{J,X_J}\dot{\phi}_I\delta\phi_I\right)\dot{\phi}_I^2 \qquad (29)$$

as the perturbation of X_I up to first order. This result is in consistency with the one derived in the MSI model [17] and see [36, 37] for a general case. The Lagrangian (28) contains a lot of coupled terms as $\delta\phi_I\dot{\delta\phi}_J$ and $\delta\phi_I\delta\phi_J$. These terms imply that the perturbations of inflaton fields depend on others during their evolution, and the equations of motion would be difficult to be solved directly. In this note we aim at making the transformation of the inflaton perturbations as to obtain approximately decoupled equations of motion for new fields. In principle this is difficult, if not impossible to realize. We will see in which case can we do such transformations and see what will happen by such a method. This will require some constraints on the slow-variation parameters. We will see this in detail for a double-field toy model.

After utilizing the equations of motion for background and taking some integration by parts, one can write the Lagrangian in a more compact form:

$$\begin{aligned}
S_2 &= \frac{1}{2}\int dtd^3xa^3\left\{(P_{I,X_I} + P_{I,X_IX_I}\dot{\phi}_I^2)\dot{\delta\phi}_I^2 - \frac{1}{a^2}P_{I,X_I}\partial_i\delta\phi_I\partial_i\delta\phi_I\right.\\
&+ \left.\mathscr{N}_{IJ}\dot{\delta\phi}_I\delta\phi_J - \mathscr{M}_{IJ}\delta\phi_I\delta\phi_J\right\} \qquad (30)
\end{aligned}$$

with $\mathscr{N}_{IJ}$ and $\mathscr{M}_{IJ}$ are time-dependent damping and mass terms,

$$\begin{aligned}
\mathscr{N}_{IJ} &= 2P_{I,IX_I}\dot{\phi}_I\delta_{IJ} - \frac{1}{H}P_{I,X_IX_I}\dot{\phi}_I^3\dot{\phi}_J, \qquad (31)\\
\mathscr{M}_{IJ} &= -P_{I,II}\delta_{IJ} + \frac{1}{H}P_{I,IX_I}P_{J,X_J}\dot{\phi}_I^2\dot{\phi}_J\\
&- \frac{1}{4H^2}P_{K,X_KX_K}\dot{\phi}_K^4 P_{I,X_I}P_{J,X_J}\dot{\phi}_I\dot{\phi}_J - \frac{1}{a^3}\left(\frac{a^3}{H}P_{I,X_I}P_{J,X_J}\dot{\phi}_I\dot{\phi}_J\right)^{\cdot}. \qquad (32)
\end{aligned}$$

This result is consistent with that obtained in multi-field inflation[23, 38, 39]. To distinguish the diagonal and off-diagonal parts of the matrices above, we define

$$\mathscr{N}_{IJ} = \mathcal{N}\delta_{IJ} + \mathcal{N}_{IJ}, \quad \mathscr{M}_{IJ} = \mathcal{M}\delta_{IJ} + \mathcal{M}_{IJ}. \qquad (33)$$

After assuming that all the slow-varying parameters defined in Sec. I are small, we have

$$\mathcal{N} = 2\epsilon\frac{e_I^2}{\epsilon_I^2}\left[\epsilon\left(1+\frac{5}{6}\epsilon\frac{\eta_I}{\epsilon_I}\right)\left(1-\frac{\eta_I}{\epsilon_I}-\frac{h_I}{e_I}\right)\left(\frac{1}{c_{sI}^2}+1\right)-\epsilon_I+\frac{\eta_I}{2}\right], \tag{34}$$

$$\mathcal{N}_{IJ} = -H\left(\frac{1}{c_{sI}^2}-1\right)e_Ie_J\left[1+\frac{1}{3}\epsilon\left(\frac{\eta_I}{\epsilon_I}+\frac{\eta_J}{\epsilon_J}\right)\right], \tag{35}$$

$$\mathcal{M}_I = 3H^2\iota_I, \tag{36}$$

$$\mathcal{M}_{IJ} = -3H^2e_Ie_J\left[1+\frac{1}{6}\sum_K\epsilon_K\left(\frac{1}{c_{sK}^2}-1\right)\right]. \tag{37}$$

Here, note that all these quantities are of order $\mathcal{O}(\epsilon)$ if $c_{sI}^2\sim\mathcal{O}(1)$. In such a case, as we will see below, the damping term has no correlations. For ordinary terms we have only preserved the leading order to slow-variation parameters, but if a term involves c_{sI}^{-2}, we preserve one higher order. This is to ensure the consistency when $c_{sI}^2\sim\mathcal{O}(1)$ fails, as in a model with multiple DBI actions. Under the condition when $c_{sI}^2\sim\mathcal{O}(\epsilon)$, all the terms should be preserved.

To investigate further we can derive the equations of motion for the fields

$$\begin{aligned}\ddot{\delta\phi}_I &+ \left\{\left[3H\left(1-\frac{2}{3}s_I\right)+(1-2c_{sI}^2)\frac{\dot{P}_{I,X_I}}{P_{I,X_I}}-2c_{sI}^2\frac{P_{I,X_IX_I}\dot{X}_I}{P_{I,X_I}}\right]\delta_{IJ}\right.\\ &+\left.\frac{c_{sI}^2}{H}\sum_J\left(\frac{1}{c_{sJ}^2}-1\right)P_{J,X_J}\dot{\phi}_I\dot{\phi}_J\right\}\dot{\delta\phi}_J\\ &+\left[\left(\frac{k^2c_{sI}^2}{a^2}+\frac{c_{sI}^2}{P_{I,X_I}}\mathcal{M}\right)\delta_{IJ}+\frac{c_{sI}^2}{P_{I,X_I}}\sum_J\mathcal{M}_{IJ}\right]\delta\phi_J=0.\end{aligned} \tag{38}$$

Note there is no summation over index "I" even if it repeats in one term. Here and throughout the paper Einstein's summation rule is not applied for the index of fields (but still valid for repetition of spatial sub/superscripts) and all the summations over capital Latin letters "$I, J, ...$" are written explicitly. In a generic case, we can rewrite the equations of motion as follows,

$$\ddot{\delta\phi}_I+3H(1+\kappa_I)\dot{\delta\phi}_I+\left(\frac{k^2c_{sI}^2}{a^2}+m_I\right)\delta\phi_I+H\sum_J\xi_{IJ}\dot{\delta\phi}_J+\sum_J m_{IJ}\delta\phi_J=0. \tag{39}$$

with

$$m_I = \frac{c_{sI}^2}{P_{I,X_I}}\mathcal{M}, \tag{40}$$

$$m_{IJ} = \frac{c_{sI}^2}{P_{I,X_I}}\sum_J\mathcal{M}_{IJ}, \tag{41}$$

$$\kappa_I = -\frac{2}{3}s_I+\frac{1}{3}\eta_I-\frac{2}{3}\eta_Ic_{sI}^2-\frac{2\epsilon}{3}\left(1-\frac{\eta_I}{\epsilon_I}-\frac{h_I}{e_I}\right)(1-c_{sI}^2), \tag{42}$$

$$\xi_{IJ} = 2\epsilon_I\frac{e_J}{e_I}c_{sI}^2\left(\frac{1}{c_{sJ}^2}-1\right), \tag{43}$$

which are also small in inflation. We see there are two different couplings in the damping and effective mass term respectively. Besides, they both depend on time via the parameters. In principle, this kind of equation is difficult to solve, if not impossible. Of course we see that the coupling terms between different fields are all very weak if we admire the slow-variation parameters to be small, which may maintein the final solutions almost the same as those of N independently evolving fields. Further more, two special cases which have specific limits of sounds of speed are most interesting. One is that all fields ϕ_I have the canonical kinetic energy, with $c_{sI} \to 1$. This is a decoupled multi-slow-roll model, which is called assisted inflation by [4] and received much focus[5–8]. In such a case, $\xi_{IJ} \sim \mathcal{O}(\epsilon^2)$ and therefore can be totally neglected in the action, while $m_{IJ} \sim \mathcal{O}(\epsilon)$. We notice that only the correlations in mass term are preserved, which are easy to manipulate. On the other hand, when all the fields are DBI-type, as is discussed in [18], c_{sI} is very small in the relativistic limit. For simplicity, we can suppose that the sound speeds are as small as ϵ, or even smaller. We will shortly confirm that, the latter case is naturally suitable for analytical calculation, while for former case one should impose further assumptions on the difference of c_s's. But whatever case it is, we see $\xi_{IJ} \sim \mathcal{O}(\epsilon)$ which is the main part of coupling, and $m_{IJ} \leq \mathcal{O}(\epsilon^{3/2})$ thus can be neglected.

One possible method is to consider the smallness of couplings, and rotate the perturbative fields in field-space into a instantaneous orthogonal basis which can generate the adiabatic perturbation which is in the direction of the trajectory of fields in background, along with $N-1$ entropy perturbations perpendicular to the direction of field trajectory. After doing that, if we suppose the coupling terms which are proportional to the derivatives of the rotating angles with respect to time of the trajectory in field-space is small at the moment of horizon crossing, we can neglect the coupling and treat the adiabatic and entropy perturbations independently as free streaming quantum fields. After some proper quantization, all the discussions are similar to a N decoupled multi-field theory[23, 38, 39]. Next if we want to consider the effect of couplings that have been omitted, one can treat such couplings as interacting vertices to the quantum fields, and see how can the fields transfer to each other in the quantum level[40]. Actually, if the fields have different speeds of sound, it is difficult to do such rotations as to make the adiabatic perturbation laid on the direction of trajectory. This is because the angle we need to take our rotation is different from that of the slope of trajectory, since sound speeds will appear in the transformation and make the "rotation" anisotropic. Further more, different sound speeds correspond to an anisotropic rescaling in the rotation of basis. We will come back and explain this assert in a two-field case by illustration later.

If we abandon the attempts to project the field into adiabatic perturbation and entropy perturbations, we may find another route to get our results. A reasonable method is to transform the fields in order to eliminate the couplings, if possible, and then treat the decoupled perturbations as free quantum fields, with a final result of curvature perturbation $\mathcal{R}$ gained by $\delta\mathcal{N}$ formalism. Before doing this, let us digress to take a look at the solution if we discard all the couplings and see the free fields limit of our theory. The result obtained here is also useful for future convenience. Under this situation, the equations contains no coupling, while the only remaining task is to find the solution of each independent field, and do quantization in ultraviolet limit. We can consider the equations of motion as

$$\ddot{\delta\phi}_I + 3H(1+\kappa_I)\dot{\delta\phi}_I + \left(\frac{k^2 c_{sI}^2}{a^2} + m_I^2\right)\delta\phi_I = 0, \tag{44}$$

where the parameter κ and m have been defined in (42) and (40). This is a little different

from the standard form of single-field inflation because of an additional term in the damping effect. We consider the fields moving in an inflationary quasi-de Sitter phase, which means the comoving time has a simple form[41]

$$\tau = -\frac{1}{Ha}\frac{1}{1-\epsilon} \tag{45}$$

And we can postulating a rescaling on the fields,

$$\begin{aligned}
\chi_I(\tau) &= z_I(\tau)^{1+\beta_I}\delta\phi_I, &(46)\\
z_I &= \frac{a\sqrt{P_{I,X_I}}}{c_{sI}^2} \approx \frac{ae_I}{c_{sI}\sqrt{2\epsilon_I}}, &(47)\\
\beta_I &= \frac{3}{2}\kappa_I - h_I + s_I + \frac{\eta_I}{2}. &(48)
\end{aligned}$$

Note that, our rescaling contains an extra power of β which is a small quantity of order $\mathcal{O}(\epsilon)$ comparing with the standard definition of Mukhanov-Sasaki variable[42] due to the small shift in the diagonal damping term of the Lagrangian. This extra power can be determined by requiring the coefficients of χ_I' term in the equation of motion vanish. The Mukhanov-Sasaki variable, as in the standard process before taking the quantization of the fields, is actually defined via the normalizer before the kinetic terms in the action(30). Therefore, we can convert the equation into a standard form which is familiar to that in single field theory:

$$\chi_I'' + \left[k^2 c_{sI}^2 - \frac{2}{\tau^2}\left(1 + \frac{9}{4}\kappa_I + \frac{3}{2}\epsilon - \frac{m_I^2}{2H^2}\right)\right]\chi_I = 0. \tag{49}$$

This is just the Bessel equation of order ν_I which will be determined below. The solution to this equation is a linear combination of two independent Bessel functions. However, to have the appropriate approximation behavior which approaches the planar wave in Bunch-Davies vacuum with correct normalization at early time $\tau \to -\infty$ [43], we need to choose the proper coefficients of the Bessel functions, which corresponds to such a state in quantum theory that it can minimize the energy density. One possible result is to choose[41, 42]

$$\chi_I(\tau) = \frac{\sqrt{-\pi\tau}}{2} e^{i\frac{\pi}{2}\left(\nu_I + \frac{1}{2}\right)} H_{\nu_I}^{(1)}(-kc_{sI}\tau), \tag{50}$$

where $H_{\nu_I}^{(1)}$ is the Hankel function of the first type, with

$$\begin{aligned}
\nu_I &= \frac{3}{2}\sqrt{1 + 2\kappa_I + \frac{4}{3}\epsilon - \frac{8}{3}c_{sI}^2\frac{\epsilon_I \iota_I}{e_I^2}} \equiv \frac{3}{2} - \sigma_I, &(51)\\
\sigma_I &\approx s_I - \epsilon + \left(c_{sI}^2 - \frac{1}{2}\right)\eta_I + \epsilon\left(1 - \frac{\eta_I}{\epsilon_I} - \frac{h_I}{e_I}\right)(1 - c_{sI}^2) + 2c_{sI}^2\frac{\epsilon_I \iota_I}{e_I^2} &(52)
\end{aligned}$$

as its order. Note that, σ_I is a small quantity of order $\mathcal{O}(\epsilon)$, which is irrelevant of the value of c_{sI} in a multi-DBI model when $c_{sI} \sim \mathcal{O}(\epsilon)$. Therefore, the amplitude of perturbation to the original field ϕ_I is

$$\mid \delta\phi_I \mid = 2^{\nu_I - 2}\frac{\Gamma(\nu_I)}{\Gamma(3/2)}\left(\frac{c_{sI}\sqrt{2\epsilon_I}}{ae_I}\right)^{2(1+\beta_I)}\frac{(-kc_{sI}\tau)^{\sigma_I}}{-\tau(kc_{sI})^{3/2}} \tag{53}$$

Again we use the fact that both ν_I and κ_I are very small, thus we get the power spectrum of each field

$$\begin{aligned}\mathcal{P}_{\delta\phi_I} &= \left(\frac{H}{2\pi}\right)^2 \frac{2}{c_{sI}} \left[1 + (\beta_I - 2\sigma_I)\ln 2 - \sigma_I \psi\left(3/2\right)\right] \left(\frac{\epsilon_I}{e_I^2}\right)^{\beta_I} \left(\frac{H}{k}\right)^{2\beta_I} (1-\epsilon)^{2(1-\sigma_I)}, \\ &\approx \left(\frac{H}{2\pi}\right)^2 \frac{2}{c_{sI}} \left(\frac{H}{k}\right)^{2\beta_I} \frac{\epsilon_I}{e_I^2} \left[1 - 2\epsilon + \beta_I \ln 2 + \sigma_I (\gamma - 2) + \beta_I \ln \frac{\epsilon_I}{e_I^2}\right], \end{aligned} \tag{54}$$

where the second approximate equality holds at the moment when the wavelength of the mode considered exits the sound horizon, i.e. $kc_{sI} = Ha$. ψ is the digamma function and relates to Euler-Mascheroni constant γ as $\psi(3/2) = -\gamma + 2 - 2\ln 2$. The dependence on the speed of sound is superficially different from that of single-field DBI inflation. But when we take an extra relationship $\dot{H} = -\sum_I X_I/c_{sI}$ which holds in DBI-type action, a new relation between the parameters c_{sI}, e_I and ϵ_I, holds as to leading order,

$$2\frac{\epsilon_I}{e_I^2} = c_{sI}. \tag{55}$$

Therefore the dependence on c_{sI} in (54) is canceled, and (54) will be coincident up to leading order with the result in [32]. By virtue of $\delta\mathcal{N}$ formalism [44, 45].

$$\zeta = \delta\mathcal{N}_e = \sum_I \mathcal{N}_{e,I}\, \delta\phi_I + \frac{1}{2}\sum_{IJ} \mathcal{N}_{e,IJ}\, \delta\phi_I \delta\phi_J + \cdots \tag{56}$$

where $\mathcal{N}_e$ is the local e-folding number along a trajectory from a spatially flat slice at a moment t_* soon after the relevant scale has passed outside the horizon during inflation, to a uniform-density slice at another moment t_c after complete reheating when ζ has become a constant. So the power spectrum curvature of perturbation ζ can be calculated by [44].

$$\mathcal{P}_\zeta = \left(\frac{H}{2\pi}\right)^2 \sum_I \mathcal{N}_{e,I}^2 \tag{57}$$

where $N_{e,I}$ can be determined via

$$H = -\sum_I \frac{\partial \mathcal{N}_e}{\partial \phi_I}\dot{\phi}_I = -2H \sum_I \frac{\partial \mathcal{N}_e}{\partial \phi_I}\frac{\epsilon_I}{e_I}\left(1 - \frac{1}{3}\epsilon\frac{\eta_I}{\epsilon_I} + \frac{2}{3}\epsilon\frac{h_I}{e_I}\right) \tag{58}$$

in a general case.

Here for simplicity, we should consider the case when the N fields moves almost in the same manner. This is proved by [4] to be a late-time attractor for some appropriate initial conditions. Later on we will demonstrate our analytical calculation for a system with different sound speeds is valid when $\epsilon_I - \epsilon_J \sim \mathcal{O}(\epsilon^2)$, which implies a field configuration like this. Under such assumptions, to order $\mathcal{O}(\epsilon)$ one have

$$\frac{\partial \mathcal{N}_e}{\partial \phi_I} = -\frac{1}{2N}\frac{e_I}{\epsilon_I}\left(1 + \frac{1}{3}\epsilon\frac{\eta_I}{\epsilon_I} - \frac{2}{3}\epsilon\frac{e_I}{h_I}\right) \tag{59}$$

in a generic case. Therefore, together with (55) we get

$$\begin{aligned}\mathcal{P}_\zeta^{(0)} &= \left(\frac{H}{2\pi}\right)^2 \frac{1}{2N^2} \sum_I \frac{1}{\epsilon_I c_{sI}} \left(\frac{H}{k}\right)^{2\beta} \\ &\quad \cdot \left[1 + \frac{2}{3}\epsilon\frac{\eta_I}{\epsilon_I} - \frac{4}{3}\epsilon\frac{e_I}{h_I} - 2\epsilon + \beta_I \ln 2 + \sigma_I(\gamma - 2) + \beta_I \ln\frac{\epsilon_I}{e_I^2}\right]. \end{aligned} \tag{60}$$

The superscript 0 denotes this power spectrum is the one for a theory neglecting couplings. We can see if all the fields have the same speed of sound, and $\epsilon_I \sim \epsilon/N$, (60) gives the results in multifield DBI inflation[23] which contains N identifying speeds of sound up to first order of slow variation parameters.

From (54), the scalar spectral index $n_{\rm s}$ is

$$n_{{\rm s}I} - 1 = -2\beta_I = -2\epsilon\left(1 - \frac{\eta_I}{\epsilon_I} - \frac{h_I}{e_I}\right)(1 - c_{{\rm s}I}^2) + 2\eta_I(1 - c_{{\rm s}I}^2) - 2h_I. \qquad (61)$$

And the contribution to spectral index of ζ mainly comes from the largest one of $n_{{\rm s}I}$'s, which is just the power spectuum of curvature perturbation.

C. Define New Fields

To consider the coupling terms as a perturbations to equations of motion (39), we could take a redefinition of inflaton fields through a linear transformation which, up to leading order in slow-variation parameters, can give a set of N decoupled equations of motion, with different coefficients appearing as new speeds of sound and new effective mass terms. In general, this is difficult to be realized. Fortunately, we are dealing with a theory that is just perturbed from a decoupled one. As the N fields have tendency to move together, they have an even smaller difference in the parameters. Intuitively the change in eigenvalue should be small if the damping and mass matrix is not far apart from a diagonal one. Actually Hoffman-Wielandt theorem in matrix analysis tells us that the smallness of "perturbation" to a diagonal matrix will cause small variation to the eigenvalues. To make it clear let us denote λ_I as the eigenvalues of a diagonal matrix corresponding to a theory with no couplings, whose diagonal entries are just the coefficients before each $\delta\phi$ term in (39), and $\hat{\lambda}_I$ to be eigenvalues of the "perturbed" matrices corresponds to the coupled coefficients (38). Then, there exists a permutation $\sigma(I)$ for the new eigenvalues $\hat{\lambda}$ that satisfies[46]

$$\left[\sum_{I=1}^{N} \mid \hat{\lambda}_{\sigma I} - \lambda_I \mid^2\right]^{1/2} \leq \parallel \delta M \parallel_2 \sim \mathcal{O}(N\epsilon). \qquad (62)$$

Here $\parallel \delta M \parallel_2$ is the Euclidean norm of a perturbation matrix δM which under our circumstance has the magnitude of order $\mathcal{O}(\epsilon)$. Thus, roughly speaking, the variation in eigenvalue is a small quantity of order less than $\mathcal{O}(\epsilon)$, if N is not very large. This fact ensures our validity of our upcoming discussions on estimating the eigenvalues after field redefinition.

Let us analyze the coupled equations under the fact that all the coupling terms are small quantities of order $\mathcal{O}(\epsilon)$. In this case we expect the solutions will differ from the decoupled ones very little. Later on we will see that, even under such case, different sound speeds will bring a fruitful physical imprint. If we impose a transformation in field space such that

$$\delta\phi_I = \sum_m R_I^m \varphi_I, \qquad (63)$$

where R is a non-singular transformation matrix. If we require the transformation to be a representation of $SO(N)$ we can define a new basis of perturbations which can be interpreted as adiabatic/entropy directions, this matrix should be set orthogonal. But here it is not

possible in our case. Our purpose of transforming the fields is to decouple. And unless for some special cases, the new fields has nothing to do with adiabatic/entropy perturbations. Actually, if all the fields have the same speed of sound, the fields redefinition (63) could be an orthogonal transformation of $SO(N)$ group. And one of the new basis would lay on the direction of tangent line of trajectory in field space, which just duplicate the result in [28]. However, for fields with different speeds of sound, we do not have such clear geometric explanation. In such a case, we will see that the transformation is even only orthogonal up to leading order $\mathcal{O}(1)$. Later on we will see how we can explain the new transformation in a field space.

To make our discussion clear we could require R is also slow-variant, which means $\dot{\mathbb{R}} \sim \mathcal{O}(\epsilon)$. This is reasonable in a quasi-de Sitter background, and its validity will be confirmed after we get the concrete form of $\mathbb{R}$. Thus, one can write the time derivative of new field transformed from $\mathbb{R}\delta\phi$ as

$$\begin{aligned} \delta\dot{\phi}_I &= R_I^m \dot{\varphi}_m + \dot{R}_I^m \varphi_m, && (64) \\ \delta\ddot{\phi}_I &\approx R_I^m \ddot{\varphi}_m + 2\dot{R}_I^m \dot{\varphi}_m. && (65) \end{aligned}$$

Substitute into the equations of motion (38), we have the equations for new variable φ. Left multiply an inverse transformation $(R^{-1})_n^I$ and take the summation over index I, we get

$$\begin{aligned} \ddot{\varphi}_n &+ \sum_{m,I,J} (R^{-1})_n^I \left[\left(3H(1+\kappa_I) + 2\dot{R}_I^m \right) \delta_{IJ} + H\xi_{IJ} R_J^m \right] \dot{\varphi}_m \\ &+ \sum_{m,I,J} (R^{-1})_n^I \left[\left(\frac{k^2 c_{sI}^2}{a^2} R_J^m + m_I^2 R_J^m + 3H(1+\kappa_I)\dot{R}_J^m \right) \delta_{IJ} \right. \\ &+ \left. m_{IJ} R_J^m + H\xi_{IJ} \dot{R}_J^m \right] \dot{\varphi}_m = 0. \end{aligned} \tag{66}$$

If we need to decouple these equations into the form of (44) and get a set of free propagating waves in ultraviolet, we should find a way to diagonalize the two coefficients in the square brackets before $\dot{\varphi}$ and φ for each equation at the same time. This is the simultaneous diagonalization of two matrices, and is in principle impossible unless the two matrices are commutative. The coefficient matrices does not commute each other since ξ_{IJ} is even asymmetric. All the diagonal matrices are commutative. So the only remaining matrices that may be non-commutative are those with m_{IJ} and ξ_{IJ} as their elements. Since their main part are diagonal elements that surely commutes, we will justify that the complete matrices with a little variation in off-diagonal parts are believed to simultaneously diagonalizable up to $\mathcal{O}(\epsilon)$. If we can find a way to do this, then we can write

$$\sum_I \left[2(R^{-1})_n^I \dot{R}_I^m + 3H(R^{-1})_n^I \kappa_I R_I^m + H \sum_J (R^{-1})_n^I \xi_{IJ} R_J^m \right] = 3HK_n \delta_n^m, \tag{67}$$

$$\begin{aligned} \sum_I \Bigg[3H(R^{-1})_n^I (1+\kappa_I) \dot{R}_I^m &+ \frac{k^2}{a^2} (R^{-1})_n^I c_{sI}^2 R_J^m \\ &+ (R^{-1})_n^I m_I^2 R_I^m + \sum_J (R^{-1})_n^I m_{IJ}^2 R_J^m \Bigg] = \left(\frac{k^2}{a^2} C_{sn} + M_n \right) \delta_n^m. \end{aligned} \tag{68}$$

Here we have neglect a $\xi\dot{\mathbb{R}}$ term since $\dot{\mathbb{R}}$ is of order $\mathcal{O}(\epsilon)$. Under such transformation the equations of motion are completely decoupled,

$$\ddot{\varphi}_n + 3H\left(1+K_n\right)\dot{\varphi} + \left(\frac{k^2}{a^2}C_{\mathrm{s}n} + M_n\right)\varphi_n = 0. \tag{69}$$

Now we use the Hoffman-Wielandt theorem which ensures us that the solution to such eigenvalue equation of a perturbed matrix is a small variation from the eigenvalue before. According to the fundamental theorem of algebra, there are N sets of such solutions to (67) and (68), each has a new speed of sound C_{s}. The $C_{\mathrm{s}n}$'s are the speeds of propagation of the new fields, akin to those of normal modes in coupled oscillation system or phonon representation in statistical system. To investigate whether the new representation will change qualitatively the physics of an decoupled system is an interesting question. For instance when $c_{\mathrm{s}I}$'s are all close to 1, can we find a mode that has a very small speed of sound $C_{\mathrm{s}n}$ that can possibly amplify the power spectrum and non-Gaussianity? We should emphasize that although this is possible, it will not appear in an weakly coupled system we are just dealing with since the reason we've mentioned above. On the contrary, if the couplings are indeed strong, the physics could be much more different. For example, in hybrid inflation[47] where the two fields have complicated coupling which is not small at the waterfall stage of inflation, there are plentiful physical phenomena far beyond our results here [48–51]. Even in a weakly coupled system we are considering, analytical calculation is difficult to do. But after we can diagonalize the damping and mass matrices and take a set of new fields φ_n which can simplify the equations of motion (38) to (69), the remaining task is very simple, just parallel to the discussion under (44) since the form of the equation is the same, only with a different definition of parameters K_n, $C_{\mathrm{s}n}$ and M_n. The most difficult part of our method is to find the transformation $\mathbb{R}$. In the next section we will calculate in detail for an example of two-field inflation and get a taste of such scheme.

III. TWO-FIELD CASE

To calculate the curvature perturbation at horizon-crossing in detail we need the quantization of the fields in ultraviolet, which requires a set of independent wave equations. As we mentioned above, to simultaneously diagonalize so many matrices in (67) and (68) seems to be an impossible task. But after we notice all the off-diagonal entries are very small and superadd an assumption that the diagonal entries are almost the same up to $\mathcal{O}(\epsilon)$, we can decouple the equations in a natural manner. In a two-field model, the fields which drive inflation are ϕ_1 and ϕ_2, and a new definition of fields provides two equations for the eigenvector of the coefficient matrix, (67)(68). We have seen that for a generic value of $c_{\mathrm{s}I}$ the off-diagonal damping and mass term will coexist. But for a model with canonical kinetic energy, $c_{\mathrm{s}I} \to 1$ and there is only coupling in mass term[52–54]. On the other hand if $c_{\mathrm{s}I}$ is small as in the multi-DBI model, the coupling in damping term survives. Let us work in the multi-DBI case, leaving aside the matrix m_{IJ} which has been proved to be up to $\mathcal{O}(\epsilon^{3/2})$, as has been discussed in the paragraph under (41). Therefore the only remaining matrix to be diagonalized is ξ_{IJ}, which has the eigenvalues

$$\begin{vmatrix} \xi_{11}-\lambda & \xi_{12} \\ \xi_{21} & \xi_{22}-\lambda \end{vmatrix} = 0. \tag{70}$$

With the eigenvalues

$$\lambda^{(1)} = 2(\epsilon_1 + \epsilon_2), \quad \lambda^{(2)} = 0. \tag{71}$$

These eigenvalues corresponds to a transformation matrix

$$\mathbb{R} = \left(1 + \frac{\epsilon_2}{\epsilon_1}\right)^{-1/2} \begin{pmatrix} 1 & -\frac{e_2 c_{\rm s1}^2}{e_1 c_{\rm s2}^2} \\ \frac{\epsilon_2 e_1 c_{\rm s2}^2}{\epsilon_1 e_2 c_{\rm s1}^2} & 1 \end{pmatrix}. \tag{72}$$

Actually the eigenvector can only determine this matrix up to two normalization constants. Here we adopt a normalization such that it will preserve the determinant to be unity. This is not important in the quantization process, but will affect the calculation of power spectrum later. We know the similarity transformation $\mathbb{R}$ will diagonalize the matrix with elements ξ_{IJ}. Next we will show that it can also diagonalize the diagonal matrices with different diagonal entries, say $\mathbb{K} = \mathrm{diag}(\kappa_1, \kappa_2)$, under some reasonable assumptions about the model. When the similarity matrix $\mathbb{R}$ operates on $\mathbb{K}$,

$$\mathbb{R}^{-1}\mathbb{K}\mathbb{R} = \frac{1}{\epsilon_1 + \epsilon_2} \begin{pmatrix} \epsilon_1\kappa_1 + \epsilon_2\kappa_2 & \epsilon_1 \frac{e_2 c_{\rm s1}^2}{e_1 c_{\rm s2}^2}(\kappa_2 - \kappa_1) \\ \epsilon_2 \frac{e_1 c_{\rm s2}^2}{e_2 c_{\rm s1}^2}(\kappa_2 - \kappa_1) & \epsilon_1\kappa_2 + \epsilon_2\kappa_1 \end{pmatrix}. \tag{73}$$

So a diagonal matrix which has different diagonal entries can remain diagonal after the similarity transformation $\mathbb{R}$if

$$\kappa_1 - \kappa_2 = \mathcal{O}(\epsilon^2) \tag{74}$$

with κ_I's and ϵ_I's in the matrix are still of order ϵ. We will see, that for a specific multi-DBI model, the assumption that $\kappa_1 - \kappa_2$ as well as $s_1 - s_2$ and $\eta_1 - \eta_2$ are of order $\mathcal{O}(\epsilon^2)$ is reasonable, since the difference between two small positive quantities of order $\mathcal{O}(\epsilon)$ must be even smaller, if the fields have the tendency to move together under an appropriate initial condition. Besides, we will suppose that the differences between other parameters like $m_1^2 - m_2^2$ and $c_{\rm s1}^2 - c_{\rm s2}^2$, are also as small as $\mathcal{O}(\epsilon^2)$ for future convenience. Note, that this do not implies $\epsilon_1 - \epsilon_2 \sim \mathcal{O}(\epsilon^2)$ which is a much more strict constraint on the evolution of fields. We will come back to this issue and make some necessary justification to a specific two-field-DBI model later. At this moment, we just emphasize that this assumption preserve the diagonal property of the diagonal matrices in (67) and (68) involving κ_I, c_{sI}^2 and m_I^2 automatically up to $\mathcal{O}(\epsilon^2)$ under a similarity transformation by any matrix $\mathbb{R}$.

Next let us check whether this matrix can make $\mathbb{R}^{-1}\dot{\mathbb{R}}$ diagonal. The entries of the matrix are

$$\begin{aligned}
(\mathbb{R}^{-1}\dot{\mathbb{R}})_{11} &= (\mathbb{R}^{-1}\dot{\mathbb{R}})_{22} = \frac{H\epsilon_2}{\epsilon_1 + \epsilon_2}\left[(h_1 - h_2) - \frac{1}{2}(\eta_1 - \eta_2) - 2(s_1 - s_2)\right], && (75) \\
(\mathbb{R}^{-1}\dot{\mathbb{R}})_{12} &= \frac{H\epsilon_1}{\epsilon_1 + \epsilon_2}\frac{e_2 c_{\rm s1}^2}{e_1 c_{\rm s2}^2}\left[(h_1 - h_2) - 2(s_1 - s_2)\right], && (76) \\
(\mathbb{R}^{-1}\dot{\mathbb{R}})_{21} &= \frac{H\epsilon_2}{\epsilon_1 + \epsilon_2}\frac{e_1 c_{\rm s2}^2}{e_2 c_{\rm s1}^2}\left[(h_1 - h_2) - (\eta_1 - \eta_2) - 2(s_1 - s_2)\right]. && (77)
\end{aligned}$$

We see all the entries are proportional to the differences among some slow-variation parameters of the two fields, which are of order $\mathcal{O}(\epsilon^2)$ as we mentioned before. Then we can assert that up to $\mathcal{O}(\epsilon)$ the time derivative of $\mathbb{R}$ multiplied by its inverse in (67) is negligible.

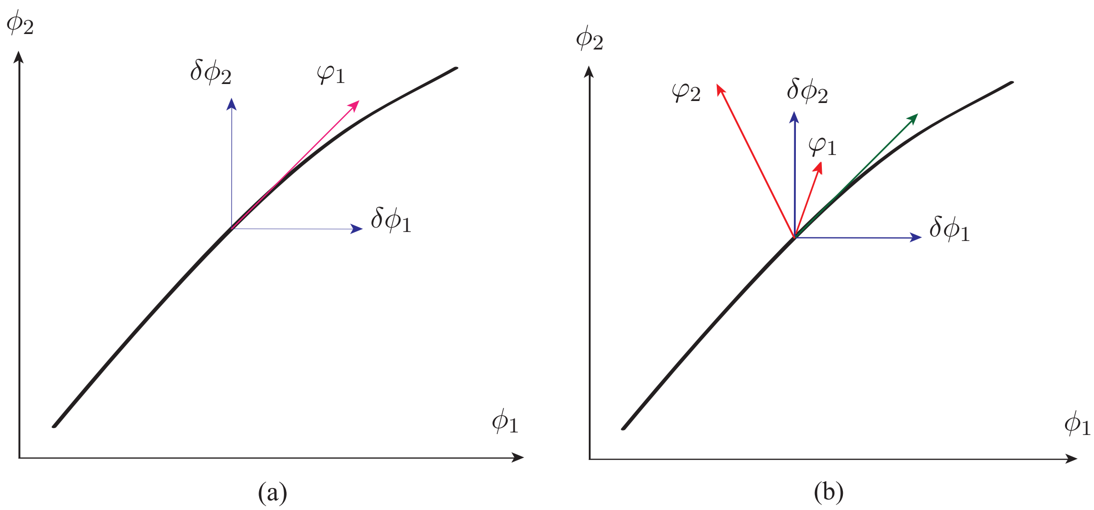

FIG. 1: The transformation $\mathbb{R}$ is depicted by a transformation in field space of the perturbations. (a) denotes the trivial case when $\epsilon_1 = \epsilon_2$, $e_1 = e_2$ and $c_{\rm s1} = c_{\rm s2}$. We see for a totally symmetric configuration $\mathbb{R}$ poses an SO(2) rotation by an angle $\pi/4$, and ϕ_1 is just the adiabatic perturbation in usual case. (b) shows a more interesting result when $\epsilon_1 = (1/\sqrt{2})\epsilon_2$, $e_1 = (1/\sqrt{2})e_2$ and $c_{\rm s1} = \sqrt{2}c_{\rm s2}$. Note this is to ensure $\dot\phi_1^2/\dot\phi_2^2 = \epsilon_1 c_{\rm s2}/(\epsilon_2 c_{\rm s1}) = 1$ such that field are moving along a diagonal trajectory in fields space. We see that $\mathbb{R}$ represents a transformation that rotates the field and rescales it at the same time with different rotating angle and anisotropic rescaling factor for two fields, i.e. the red arrow denoting the basis of new fields are not orthogonal, and the new fieds φ can not preserve the unitarity.

We will pause for a while and see the possible geometric explanation of transformation $\mathbb{R}$. We have seen that it is only orthogonal up to $\mathcal{O}(\epsilon)$ under the assumption that $\epsilon_1 - \epsilon_2 \sim \mathcal{O}(\epsilon^2)$. When $c_{\rm s1} = c_{\rm s2}$ as is the case of ordinary assisted inflation, one have that the transformation $\mathbb{R}$ is just rotating the basis by $\pi/4$ and φ_2 is just the adiabatic perturbation with φ_1 the entropy perturbation which is negligible, as is depicted in Fig.1(a). A more interesting case is the theory with different speeds of sound, as in (b).

After the transformation, one can find that two equations (38) are decoupled, which is just (69),

$$\ddot\varphi_n + 3H\left(1 + K_n\right)\dot\varphi + \left(\frac{k^2}{a^2}C_{\rm sn} + M_n\right)\varphi_n = 0$$

with

$$K_1 = \frac{\epsilon_1\kappa_1 + \epsilon_2\kappa_2}{\epsilon_1 + \epsilon_2} + \frac{2}{3}(\epsilon_1 + \epsilon_2), \quad K_2 = \frac{\epsilon_2\kappa_1 + \epsilon_1\kappa_2}{\epsilon_1 + \epsilon_2}, \tag{78}$$

$$C_{\rm s1}^2 = \frac{\epsilon_1 c_{\rm s1}^2 + \epsilon_2 c_{\rm s2}^2}{\epsilon_1 + \epsilon_2}, \quad C_{\rm s2}^2 = \frac{\epsilon_2 c_{\rm s1}^2 + \epsilon_1 c_{\rm s2}^2}{\epsilon_1 + \epsilon_2}, \tag{79}$$

$$M_1 = \frac{\epsilon_1 m_1^2 + \epsilon_2 m_2}{\epsilon_1 + \epsilon_2}, \quad M_2 = \frac{\epsilon_2 m_1^2 + \epsilon_1 m_2}{\epsilon_1 + \epsilon_2}. \tag{80}$$

To go further let us study the change of the normalizer z_I under the similarity transformation by $\mathbb{R}$. Note that, for example, when a normalizer matrix $\mathbb{Z}$, which is also diagonal, is multiplied by another matrix, say $\mathbb{X}$, the product is transformed under $\mathbb{R}$ by

$$\mathbb{R}^{-1}\mathbb{Z}\mathbb{X}\mathbb{R} = \mathbb{R}^{-1}\mathbb{Z}\mathbb{R}\mathbb{R}^{-1}\mathbb{X}\mathbb{R}, \tag{81}$$

which is just multiplying each diagonal entries after the similarity transformation respectively. Denote by Z_i the diagonal entries of $\mathbb{R}^{-1}\mathbb{Z}\mathbb{R}$,

$$
\begin{aligned}
Z_1(\tau) &= \frac{a(\tau)}{\sqrt{2}}\frac{c_{s2}e_1\sqrt{\epsilon_1}+c_{s1}e_2\sqrt{\epsilon_2}}{c_{s1}c_{s2}(\epsilon_1+\epsilon_2)}, &(82)\\
Z_2(\tau) &= \frac{a(\tau)}{\sqrt{2}}\frac{c_{s1}e_2\epsilon_2\epsilon_1^{-1/2}+c_{s2}e_1\epsilon_1\epsilon_2^{-1/2}}{c_{s1}c_{s2}(\epsilon_1+\epsilon_2)}. &(83)
\end{aligned}
$$

After doing this, we can turn to the same process after equation (69). Then we define new field variables

$$
\begin{aligned}
\chi_n(\tau) &= Z_n(\tau)^{1+B_n}\varphi_n, &(84)\\
B_1 &= \frac{3}{2}K_1+\frac{c_{s2}e_1\sqrt{\epsilon_1}\,\triangle_1+c_{s1}e_2\sqrt{\epsilon_2}\triangle_2}{c_{s2}e_1\sqrt{\epsilon_1}+c_{s1}e_2\sqrt{\epsilon_2}}, &(85)\\
B_2 &= \frac{3}{2}K_2+\frac{c_{s2}e_1\epsilon_1\epsilon_2^{-1/2}\,\triangle_1+c_{s1}e_2\epsilon_2\epsilon_1^{-1/2}\triangle_2}{c_{s1}e_2\epsilon_2\epsilon_1^{-1/2}+c_{s2}e_1\epsilon_1\epsilon_2^{-1/2}}, &(86)\\
\triangle_n &= -\frac{1}{2}(\eta-\eta_n)+h_n-s_n, &(87)
\end{aligned}
$$

which satisfy the equations of motion

$$
\chi_n''+\left[k^2C_{sn}^2-\frac{2}{\tau^2}\left(1+\frac{9}{4}K_n+\frac{3}{2}\epsilon+\eta-\frac{1}{2}\triangle_n-\frac{M_n^2}{2H^2}+\frac{M_n^2}{H^2}(\epsilon-\eta)\right)\right]\chi_I=0. \qquad (88)
$$

Now the equations have been decoupled. For each equation, the solution is a Hankel function with order $3/2-\varsigma_n$, where

$$
\varsigma_n=-K_n-\frac{2}{3}\epsilon-\frac{8}{9}\eta+\frac{2}{9}\frac{M_n^2}{H^2}, \qquad (89)
$$

Finally, we obtain the power spectra of the new fields φ_i as follows,

$$
\begin{aligned}
\mathcal{P}_{\varphi_1} &= \left(\frac{H}{2\pi}\right)^2\frac{2}{C_{s1}}\left(\frac{kC_{s1}}{Ha}\right)^{2\varsigma_1}\frac{C_{s2}^2(\epsilon_1+\epsilon_2)^2}{(C_{s2}e_1\sqrt{\epsilon_1}+C_{s1}e_2\sqrt{\epsilon_2})^2}\\
&\quad\cdot\left[1-2\epsilon-2\varsigma_1\left(\ln 2+\psi\left(3/2\right)\right)+(3K_1-2\triangle_1)\ln\frac{C_{s1}C_{s2}(\epsilon_1+\epsilon_2)}{C_{s2}e_1\sqrt{\epsilon_1}+C_{s1}e_2\sqrt{\epsilon_2}}\right], &(90)\\
\mathcal{P}_{\varphi_2} &= \left(\frac{H}{2\pi}\right)^2\frac{2}{C_{s2}}\left(\frac{kC_{s2}}{Ha}\right)^{2\varsigma_2}\frac{C_{s1}^2(\epsilon_1+\epsilon_2)^2}{(C_{s1}e_2\epsilon_2\epsilon_1^{-1/2}+C_{s2}e_1\epsilon_1\epsilon_2^{-1/2})^2}\\
&\quad\cdot\left[1-2\epsilon-2\varsigma_2\left(\ln 2+\psi\left(3/2\right)\right)+(3K_2-2\triangle_2)\ln\frac{C_{s1}C_{s2}(\epsilon_1+\epsilon_2)}{C_{s1}e_2\epsilon_2\epsilon_1^{-1/2}+C_{s2}e_1\epsilon_1\epsilon_2^{-1/2}}\right]. &(91)
\end{aligned}
$$

The power spectra above are calculated at the moment when the wavelength exceeds the sound horizon, respectively. When one field, say φ_1 have exceeded the sound horizon, the power spectrum of φ_1 is frozen, and so the curvature perturbation calculated later should involve the value of $\mathcal{P}_{\varphi_1}$ calculated at t_{*1} which satisfies $Ha(t_{*1})=kC_{s1}$. The power spectrum

of curvature perturbation should be calculated at the moment when the wavelength has exceeded all the sound horizons corresponding to different φ's. The power spectra are nearly scale-invariant, with $-2\varsigma_n$ as their spectral index. The spectral index of curvature perturbation ζ is thus $-2\varsigma_n$ which has the largest absolute value.

Using $\delta\mathcal{N}$ formalism, we can calculate the curvature perturbation after the wavelength being stretched out of the largest sound horizon. Note that to leading order

$$\zeta = \delta\mathcal{N}_e = \sum_I \mathcal{N}_{e,I}\,\delta\phi_I = \sum_I \mathcal{N}_{e,I}\,R_I^m \varphi_m. \tag{92}$$

There is no correlation between different φ's since they are independent kinetic modes as we defined. After using the additional relation (55) we can substitute e_n in the expression by $\sqrt{2\epsilon_n}/c_{sn}$. Therefore we can simplify the final result

$$\mathcal{P}_\zeta \;=\; \left(\mathcal{N}_{e,1}^{\,2} + \frac{\epsilon_2 c_{s2}^5}{\epsilon_1 c_{s1}^5}\mathcal{N}_{e,2}^{\,2}\right)\mathcal{P}_{\varphi_1} + \left(\frac{\epsilon_2 c_{s1}^5}{\epsilon_1 c_{s2}^5}\mathcal{N}_{e,1}^{\,2} + \mathcal{N}_{e,2}^{\,2}\right)\mathcal{P}_{\varphi_2}. \tag{93}$$

The derivatives of the e-folding number can be calculated in a specific model in detail, for instance in a model with standard kinetic energy and separable potential[55]. Since we only want to emphasize the effects caused by different sound speeds, again we use the assumption that both fields are moving in the same manner, i.e. in the diagonal trajectory in field space as in (b) of Fig.1, and (59) holds. This kind of motion is nearly adiabatic, which will always suppress the isocurvature perturbation inside the sound horizon. The final result is a little lengthy and instead of writing it here, we would rather focus on the dependence on different sound speeds by calculating $\mathcal{P}_\zeta/\mathcal{P}_\zeta^{(0)}$. By defining $c_{s1} = x c_{s2}$ one gets

$$C_{s1}^2 \sim x, \quad C_{s2}^2 \sim \frac{1+x^2}{1+x}. \tag{94}$$

Now the power spectrum degenerates to

$$\begin{aligned}\frac{\mathcal{P}_\zeta}{\mathcal{P}_\zeta^{(0)}} \;&=\; \frac{1}{2}\left(1+\frac{1}{x^4}\right)\frac{1}{\sqrt{x}}\left(\frac{1}{x^2}\sqrt{\frac{x}{1+x}} + \frac{x}{\sqrt{1+x^3}}\right)^{-2} \\ &+\; \frac{1}{2}\left(1+x^6\right)\sqrt{\frac{(1+x)^3}{1+x^3}}\left(x + \frac{1}{x^2}\sqrt{\frac{1+x^3}{1+x}}\right)^{-2},\end{aligned} \tag{95}$$

which now just contains quantities up to leading order, and is in coincidence with (60) when $x = 1$, i.e. $c_{s1} = c_{s2}$. This normalized power spectrum seems to be divergent when $x \to 0$ or $x \to \infty$. However, since we require $c_{s1}^2 - c_{s2}^2 \sim \mathcal{O}(\epsilon^2)$, the corresponding physical region is the neighborhood of $x \sim \mathcal{O}(1)$. We depict the dependence on x in Fig.2.

We can see from Fig.2 that, there is a minimum of $\mathcal{P}_\zeta$ when $c_{s1} = 0.85 c_{s2}$, where $\mathcal{P}_\zeta = 0.90\mathcal{P}_\zeta^{(0)}$. But the most interesting aspect is that if one of the speed of sound is a few times larger than the other (with themselves still as small as $\mathcal{O}(\epsilon^2)$ so as to satisfy our condition required before), the power spectrum of curvature perturbation increases magnificently. We see from Fig.2 and (95) that if $x > 1$, the contribution from φ_2 exceeds that from φ_1, and vis visa if $x < 1$. Therefore we have encountered an interesting feature different from our intuition out of a theory without coupling, that the field with smaller speed of sound will

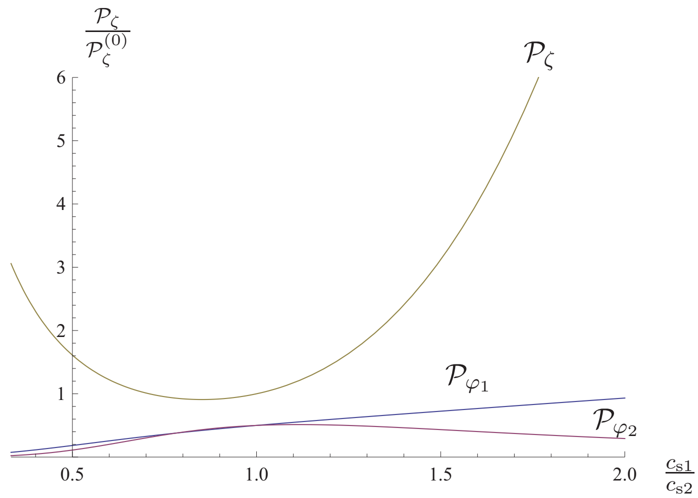

FIG. 2: The dependence on c_s of the power spectrum. Both the power spectra of φ are normalized by $(H/2\pi)^2$. and $\mathcal{P}_\zeta$ is normalized by the un-coupled one (60). There is severe divergence of $\mathcal{P}_\zeta$ when one of the sound speed vanish, but the only valid domain under our assumption is the case when $c_{s1} \sim c_{s2}$, i.e. in the neighborhood of $x \sim 1$. We see the amplification by the reciprocal sound speed is much larger than that in a trivial $x = 1$ case of ordinary unique-speed model.

not in general contribute more to the curvature perturbation. For instance, if $c_{s1} > c_{s2}$, it is the power spectrum from auxiliary field φ_2 which contributes more to the final curvature perturbation, not $\delta\phi_1$. We also see that if c_{s1}/c_{s2} deviates from 1, the power spectrum of curvature perturbation is amplified significantly, compared with the one without couplings. This just tells us that the effect given by the coupling terms may be large and can not be neglect in previous calculations. Since the power spectrum of curvature perturbation can be observed to rather high accuracy, this result actually tells us that we can loosen the constraints on the slow-variation parameters of individual inflaton even further.

IV. CONCLUSIONS AND DISCUSSION

In the frame of inflationary cosmology involving multiple inflaton fields, it is possible to allow these fields possess their own sound speed parameters which characterize the propagation of field fluctuations correspondingly. It is important to study how these modes are related to the cosmological perturbation we may observe in CMB experiments. In the present paper we analyzed the dynamics of field fluctuations in a general scenario of MSI. Assuming no direct coupling terms among the inflaton fields, we noticed that these perturbation modes are generically coupled both through their effective mass terms and cosmic damping terms. Fortunately, we found that if these couplings are weak, the perturbation theory can be treated as a combination of normal modes, by which the curvature perturbation at the horizon-crossing can be calculated in detail up to leading order of slow-variation parameters via $\delta\mathcal{N}$ formalism for a specific model.

Specifically we studied a model of multi-speed inflation involving two DBI fields with their sound speeds being small and slow-varying. Our analysis showed that in the relativistic limit, the coupling between two field fluctuations is mainly contributed by the damping terms in their perturbation equations. We further studied the curvature perturbation in this model,

and verified their primordial power spectra are nearly scale-invariant. This has interesting implications to the curvaton scenario[56], which suggests a light field in inflationary phase could seed entropy perturbation and convert it into curvature perturbation at the end of inflation[56–60][1]. Namely, as shown in [62–64], a light DBI field can realize the curvaton scenario and generate sizable non-Gaussianities of local and equilateral types after suitable fine-tuning on the model parameters. Our study also showed that, when the sound speed parameters for the inflaton fields are not equal, the perturbation modes would get frozen at different sound horizons. An important phenomenon is that the curvature perturbation of MSI could obtain an enhancement which depends on the ratio of the sound speeds.

The scenario of MSI is a very important branch of various models of multiple field inflation. In recent years, the inflationary models involving multiple field components have been studied extensively in the literature, namely, the analysis on the dynamics of N-flation [65]; multiple field inflation with particle decays [66]; the N-flation model in the frame of braneworld [67]; the model of staggered Nflation in the stringy landscape [68]; dynamics of the model of coupled N-flation with canonical fields [69]; the inflation model constructed by a DBI field coupled to radiation [70]; and see [71, 72] for comprehensive reviews on this field. It has been well realized that, for the model of multi-field inflation, when the sound speeds for the fields are the same, the field fluctuations are able to be rotated orthogonally to a basis in which the curvature and entropy fluctuations decouple explicitly, at least up to leading order. However, when we relax the scenario by allowing the sound speed parameters are different, our analysis implied that the rotation of the field fluctuations are no longer orthogonal but we still are able to find a transformation to decouple the fields under some conditions and obtain the curvature perturbation via $\delta\mathcal{N}$ formalism.

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